

Notes on the Chameleon Brans-Dicke Gravity

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Abstract

We consider a generalized Brans-Dicke model in which the scalar field has a potential function and is also allowed to couple non-minimally with the matter sector. This anomalous gravitational coupling can in principle avoid the model to pass local gravity experiments. One then usually assumes that the scalar field has a chameleon behavior in the sense that it acquires a density-dependent effective mass. While it can take a small effective mass in cosmological (low-density environment) scale, it has a sufficiently heavy mass in Solar System (large-density environment) and then hides gravity tests. We will argue that such a chameleon behavior can not be generally realized and depends significantly on the forms attributed to the potential and the coupling functions.

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1 Introduction

One of the approaches to explain accelerating expansion of the universe is to attribute this phenomenon to some modifications of general relativity. Such modified gravity models can be obtained in different ways. For instance, one can replace the Ricci scalar in the Einstein-Hilbert action by some functions $f(R)$ (for a review see, e.g., [1] and references therein), or by considering a scalar partner for the metric tensor for describing geometry of spacetime, the so-called scalar-tensor gravity. The prototype of the latter is Brans-Dicke (BD) theory [2] which its original motivation was the search for a theory containing Mach's principle. As the simplest and best-studied generalization of general relativity, it is natural to think about the BD scalar field as a possible candidate for producing cosmic acceleration without invoking auxiliary fields or exotic matter systems. In fact, there have been many attempts to show that

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BD model can potentially explain the cosmic acceleration. It is shown that this theory can actually produce a non-decelerating expansion for low negative values of the BD parameter [3]. Unfortunately, this conflicts with the lower bound imposed on this parameter by solar system experiments [4]. Due to this difficulty, some authors propose modifications of the BD model such as introducing some potential functions for the scalar field [5], or considering a field-dependent BD parameter [6] without resolving the problem.

In a general scalar-tensor theory there is a non-minimal coupling between the scalar field and Ricci scalar while the former minimally couples with the matter sector. In other terms, there is no an explicit coupling between the scalar field and matter systems in Jordan frame representation. In BD theory, in its original form, the motivation for such a minimal coupling was to keep the theory in accord with the weak equivalence principle [2]. There has recently been a tendency in the literature to go a step further and consider a non-minimal coupling between the scalar field and matter systems as well by introducing an arbitrary function of the scalar field as a coupling function [7] [8] [9]. In these models, the scalar field is regarded as a chameleon field and it is assumed that it can be heavy enough in the environment of the laboratory tests so that the local gravity constraints suppressed. Meanwhile, it can be light enough in the low-density cosmological environment to be considered as a candidate for dark energy. The large scale behavior of this chameleon BD theory has been already studied in a different work [9]. It is clear that these different behaviors in small and large scales depend crucially on the shapes of the potential and the coupling functions since both functions contribute to the effective mass or compton wavelength of the scalar field. In the present work we will study behavior of the theory in small scales. We will study the conditions that should hold in order that the theory passes solar system experiment. We will show that the conditions can not be satisfied for some usual forms of the potential and the coupling functions.

2 The model

We consider the action functional

$$S = \frac{1}{16\pi} \int d^4x \sqrt{-\bar{g}} \{ \phi \bar{R} - \frac{\omega}{\phi} \bar{g}^{\mu\nu} \bar{\nabla}_\mu \phi \bar{\nabla}_\nu \phi - V(\phi) + 16\pi f(\phi) L_m \} \quad (1)$$

where R is the Ricci scalar, ϕ is the BD scalar field, $V(\phi)$ and $f(\phi)$ are some analytic functions. Here the matter Lagrangian density, denoted by L_m , is coupled with ϕ via the function $f(\phi)$. This allows a non-minimal interaction between the matter system and ϕ . Taking $f(\phi) = 1$, we return to the BD action with a potential function $V(\phi)$.

A conformal transformation

$$\bar{g}_{\mu\nu} \rightarrow g_{\mu\nu} = \Omega^2 \bar{g}_{\mu\nu} \quad (2)$$

with $\Omega = \sqrt{G\phi}$ brings the above action into the Einstein frame [10] [11]. Then a scalar field redefinition

$$\varphi(\phi) = \sqrt{\frac{2\omega + 3}{16\pi G}} \ln\left(\frac{\phi}{\phi_0}\right) \quad (3)$$

with $\phi_0 \sim G^{-1}$, $\phi > 0$ and $\omega > -\frac{3}{2}$ transforms the kinetic term of the scalar field into a canonical form. In terms of the variables $(g_{\mu\nu}, \varphi)$ the action (1) takes the form

$$S_{EF} = \int d^4x \sqrt{-g} \left\{ \frac{R}{16\pi G} - \frac{1}{2} g^{\mu\nu} \nabla_\mu \varphi \nabla_\nu \varphi - U(\varphi) + \exp\left(-8\sqrt{\frac{\pi G}{2\omega+3}}\varphi\right) f(\varphi) L_m \right\} \quad (4)$$

Here ∇_μ is the covariant derivative of the rescaled metric $g_{\mu\nu}$. The Einstein frame potential is given by

$$U(\varphi) = V(\phi(\varphi)) \exp(-\sigma\varphi/M_p) \quad (5)$$

in which $\sigma = 8\sqrt{\frac{\pi}{2\omega+3}}$ and $M_p = G^{-1/2}$.

Varying the action (4) with respect to the metric $g_{\mu\nu}$ and φ yields the field equations,

$$G_{\mu\nu} = 8\pi G (h(\varphi) T_{\mu\nu}^m + T_{\mu\nu}^\varphi) \quad (6)$$

$$\square\varphi - U'(\varphi) = -h'(\varphi) L_m \quad (7)$$

where

$$T_{\mu\nu}^\varphi = (\nabla_\mu \varphi \nabla_\nu \varphi - \frac{1}{2} g_{\mu\nu} \nabla_\alpha \varphi \nabla^\alpha \varphi) - U(\varphi) g_{\mu\nu} \quad (8)$$

$$T_{\mu\nu}^m = \frac{-2}{\sqrt{-g}} \frac{\delta(\sqrt{-g} L_m)}{\delta g^{\mu\nu}} \quad (9)$$

Here $h(\varphi) = e^{-\sigma\varphi/M_p} f(\varphi)$, $T^m = g^{\mu\nu} T_{\mu\nu}^m$ and prime indicates differentiation with respect to φ . Due to explicit coupling of the matter system with the scalar field, the stress-tensor $T_{\mu\nu}^m$ is not divergence free. This can be seen by applying the Bianchi identities $\nabla^\mu G_{\mu\nu} = 0$ to (6), which leads to

$$\nabla^\mu T_{\mu\nu}^m = (L_m - T^m) \nabla_\nu \ln h(\varphi) \quad (10)$$

As it is clear from (10), details of the energy exchange between matter and φ depends on the explicit form of the matter Lagrangian density L_m . Here we consider a perfect fluid energy-momentum tensor as a matter system

$$T_{\mu\nu}^m = (\rho_m + p_m) u_\mu u_\nu + p_m g_{\mu\nu} \quad (11)$$

where ρ_m and p_m are energy density and pressure, respectively. The four-velocity of the fluid is denoted by u_μ .

There are different choices for the perfect fluid Lagrangian density which all of them leads to the same energy-momentum tensor and field equations in the context of general relativity [12] [13]. The two Lagrangian densities that have been widely used in the literature are $L_m = p_m$ and $L_m = -\rho_m$ [14] [15] [16]. For a perfect fluid that does not couple explicitly to the curvature (i.e., for $f(\varphi) = 1$), the two Lagrangian densities $L_m = p_m$ and $L_m = -\rho_m$ are perfectly equivalent, as discussed in [15] [16]. However, in the model presented here the expression of L_m enters explicitly the field equations and all results strongly depend on the choice of L_m . In fact, it is shown that there is a strong debate about equivalency of different expressions attributed to the Lagrangian density of a coupled perfect fluid [17]. Here we take $L_m = -\rho_m$ for the lagrangian density.

We consider $T_{\mu\nu}^m$ as the stress-tensor of dust. In a static and spherically symmetric spacetime the equation (7) gives

$$\frac{d^2\varphi}{dr^2} + \frac{2}{r} \frac{d\varphi}{dr} = \frac{dV_{eff}(\varphi)}{d\varphi} \quad (12)$$

where r is distance from center of the symmetry in the Einstein frame and

$$V_{eff}(\varphi) = \{V(\varphi) + \rho_m f(\varphi)\} e^{-\sigma\varphi/M_p} \quad (13)$$

To proceed further, we should have the explicit form of the functions $V(\varphi)$ and $f(\varphi)$. In the present work, we take $f(\varphi)$ as an exponential function $f(\varphi) = e^{l_2\varphi/M_p}$ with l_2 being a constant dimensionless parameter. For the potential function, we will consider the following cases :

2.1 Exponential potentials

We first consider the potential $V(\varphi) = e^{l_1\varphi/M_p}$. In this case, the effective potential takes the form

$$V_{eff}(\varphi) = e^{(l_1-\sigma)\varphi/M_p} + \rho_m e^{\beta\varphi/M_p} \quad (14)$$

where $\beta = l_2 - \sigma$. One can find solutions for $V'_{eff}(\varphi) = 0$, which gives

$$\varphi_{min} = \frac{M_p}{l_1 - l_2} \ln\left[\frac{\beta\rho_m}{(\sigma - l_1)}\right] \quad (15)$$

This is a local minimum if the following condition is satisfied

$$V''_{eff}(\varphi_{min}) = \frac{\rho_m}{M_p^2} e^{\beta\varphi_{min}/M_p} \beta(l_2 - l_1) > 0 \quad (16)$$

For a spherically symmetric body with a radius r_c and constant energy densities ρ_{in} ($r < r_c$) and ρ_{out} ($r > r_c$), there is a thin-shell condition [18]

$$\frac{\Delta r_c}{r_c} = \frac{\varphi_{min(out)} - \varphi_{min(in)}}{6\beta M_p \Phi_c} \ll 1 \quad (17)$$

where $\Phi_c = M_c/8\pi M_p^2 r_c$ is the Newtonian potential at $r = r_c$ with M_c being the mass of the body. In this expression, $\varphi_{min(in)}$ and $\varphi_{min(out)}$ denote the field values at two minima of the effective potential $V_{eff}(\varphi)$ inside and outside the object, respectively. They must clearly satisfy $V'_{eff}(\varphi_{min(in)}) = 0$ and $V'_{eff}(\varphi_{min(out)}) = 0$. In this case, equation (12) with some appropriate boundary conditions gives the field profile outside the object [18]

$$\varphi(r) = -\frac{\beta}{4\pi M_p} \frac{3\Delta r_c}{r_c} \frac{M_c e^{-m_{out}(r-r_c)}}{r} + \varphi_{min(out)} \quad (18)$$

As usual, masses of small fluctuations about the minima are given by $m_{in} = [V''_{eff}(\varphi_{min(in)})]^{1/2}$ and $m_{out} = [V''_{eff}(\varphi_{min(out)})]^{1/2}$ which depend on ambient matter density. A region with large mass density corresponds to a heavy mass field while regions with low mass density corresponds

to a field with lighter mass. In this way it is possible for the mass field to take sufficiently large values near massive objects in the Solar System scale and to hide the local tests.

1. *Thin – shell condition*

In the chameleon mechanism, the chameleon field is trapped inside large and massive bodies and its influence on the other bodies is only due to a thin-shell near the surface of the body. The criterion for this thin-shell condition is given by (17). If we combine (15) and (17) we obtain

$$\frac{\Delta r_c}{r_c} = \frac{1}{6\Phi_c\beta(l_1 - l_2)} \ln \frac{\rho_{out}}{\rho_{in}} \quad (19)$$

In weak field approximation, the spherically symmetric metric in the Jordan frame is given by

$$ds^2 = -[1 - 2X(\bar{r})]dt^2 + [1 + 2Y(\bar{r})]d\bar{r}^2 + \bar{r}^2 d\Omega^2 \quad (20)$$

where $X(\bar{r})$ and $Y(\bar{r})$ are some functions of \bar{r} . There is a relation between r and \bar{r} so that $r = \Omega\bar{r}$ with $\Omega = e^{\sigma\varphi/4M_p}$. Note that local gravity experiments constrain the BD parameter so that $\omega > 3500$ [4], or equivalently $\sigma < 0.17$. For φ/M_p not much greater than unity, it implies that $\bar{r} \approx r$. Assuming $m_{out} r \ll 1$, namely that the Compton wavelength m_{out}^{-1} is much larger than Solar System scales, the chameleon mechanism gives for the post-Newtonian parameter γ [19]

$$\gamma = \frac{3 - \frac{\Delta r_c}{r_c}}{3 + \frac{\Delta r_c}{r_c}} \simeq 1 - \frac{2}{3} \frac{\Delta r_c}{r_c} \quad (21)$$

We can now apply (19) on the Earth and obtain the condition that the Earth has a thin-shell. To do this, we assume that the Earth is a solid sphere of radius $R_e = 6.4 \times 10^8 \text{ cm}$ and mean density $\rho_e \sim 10 \text{ gr/cm}^3$. We also assume that the Earth is surrounded by an atmosphere with homogenous density $\rho_a \sim 10^{-3} \text{ gr/cm}^3$ and thickness 100 km . In this case, (19) takes the form

$$\frac{\Delta R_e}{R_e} = \frac{1}{6\Phi_e\beta(l_1 - l_2)} \ln \frac{\rho_a}{\rho_e} \quad (22)$$

in which $\Phi_e = 6.95 \times 10^{-10}$ is Newtonian potential on surface of the Earth [20]. The tightest Solar System constraint on γ comes from Cassini tracking which gives $|\gamma - 1| < 2.3 \times 10^{-5}$ [4]. This together with (21) and (22) yields

$$|(l_1 - l_2)(l_2 - \sigma)| > 10^{14} \quad (23)$$

2. *Equivalence principle*

We now consider constraints coming from possible violation of weak equivalence principle. We assume that the Earth, together with its surrounding atmosphere, is an isolated body and neglect the effect of the other compact objects such as the Sun, the Moon and the other planets. Far away the Earth, matter density is modeled by a homogeneous gas with energy density $\rho_G \sim 10^{-24} \text{ gr/cm}^3$. To proceed further, we first consider the condition that the atmosphere of the Earth satisfies the thin-shell condition [18]. If the atmosphere has a thin-shell the thickness of the shell (ΔR_a) must be clearly smaller than that of the atmosphere itself, namely $\Delta R_a < R_a$, where R_a is the outer radius of the atmosphere. If we take thickness

of the shell equal to that of the atmosphere itself $\Delta R_a \sim 10^2 \text{ km}$ we obtain $\frac{\Delta R_a}{R_a} < 1.5 \times 10^{-2}$. It is then possible to relate $\frac{\Delta R_e}{R_e} = \frac{\varphi_{\min(a)} - \varphi_{\min(e)}}{6\beta M_p \Phi_e}$ and $\frac{\Delta R_a}{R_a} = \frac{\varphi_{\min(G)} - \varphi_{\min(a)}}{6\beta M_p \Phi_a}$ where $\varphi_{\min(e)}$, $\varphi_{\min(a)}$ and $\varphi_{\min(G)}$ are the field values at the local minimum of the effective potential in the regions $r < R_e$, $R_a > r > R_e$ and $r > R_a$ respectively. Using the fact that newtonian potential inside a spherically symmetric object with mass density ρ is $\Phi \propto \rho R^2$, one can write $\Phi_e = 10^4 \Phi_a$ where Φ_e and Φ_a are Newtonian potentials on the surface of the Earth and the atmosphere, respectively. This gives $\Delta R_e/R_e \approx 10^{-4} \Delta R_a/R_a$. With these results, the condition for the atmosphere to have a thin-shell is

$$\frac{\Delta R_e}{R_e} < 1.5 \times 10^{-6} \quad (24)$$

The tests of equivalence principle measure the difference of free-fall acceleration of the Moon and the Earth towards the Sun. The constraint on the difference of the two acceleration is given by [4]

$$\frac{|a_m - a_e|}{a_N} < 10^{-13} \quad (25)$$

where a_m and a_e are acceleration of the Moon and the Earth respectively and a_N is the Newtonian acceleration. The Sun and the Moon are all subject to the thin-shell condition [18] and the field profile outside the spheres are given by (18) with replacement of corresponding quantities. The accelerations a_m and a_e are then given by [18]

$$a_e \approx a_N \left\{ 1 + 18\beta^2 \left(\frac{\Delta R_e}{R_e} \right)^2 \frac{\Phi_e}{\Phi_s} \right\} \quad (26)$$

$$a_m \approx a_N \left\{ 1 + 18\beta^2 \left(\frac{\Delta R_e}{R_e} \right)^2 \frac{\Phi_e^2}{\Phi_s \Phi_m} \right\} \quad (27)$$

where $\Phi_e = 6.95 \times 10^{-10}$, $\Phi_m = 3.14 \times 10^{-11}$ and $\Phi_s = 2.12 \times 10^{-6}$ are Newtonian potentials on the surfaces of the Earth, the Moon and the Sun, respectively [20]. This gives a difference of free-fall acceleration

$$\frac{|a_m - a_e|}{a_N} \approx \beta^2 \left(\frac{\Delta R_e}{R_e} \right)^2 \quad (28)$$

Combining this with (25) results in

$$\beta \frac{\Delta R_e}{R_e} < 10^{-7} \quad (29)$$

Taking this as the constraint coming from violation of equivalence principle and combining with (22), we obtain

$$|l_1 - l_2| > 10^{16} \quad (30)$$

The constraints (23) and (30) imply that $l_2 - \sigma \sim 10^{-2}$ and one can take $l_2 \sim \sigma < 0.17$. This means that local experiments are satisfied only for extremely large values of l_1 . Since $l_1 > l_2$, the condition (16) and the expression (15) require that $\beta < 0$. Thus the second term in the effective potential is a decreasing function while $U'(\varphi) > 0$. This ensures that $V_{eff}(\varphi)$ does exhibit a local minimum corresponding to an effective mass.

2.2 Power-law potentials

In this case, we take $V(\varphi) = V_0\varphi^{l_3}$ with l_3 being a dimensionless constant parameter. The effective potential takes then the form

$$V_{eff}(\varphi) = V_0\varphi^{l_3}e^{-\sigma\varphi/M_p} + \rho_m e^{\beta\varphi/M_p} \quad (31)$$

For $\varphi \ll M_p$, one can write $e^{l_2\varphi/M_p} \approx 1 + \frac{l_2\varphi}{M_p}$. Then one can find solutions for $V'_{eff}(\varphi) = 0$, which gives

$$V_0\varphi^{l_3}\left(\frac{l_3}{\varphi} - \frac{\sigma}{M_p}\right) - \frac{\beta}{M_p}\rho_m\left(1 + \frac{l_2\varphi}{M_p}\right) = 0 \quad (32)$$

Since $\sigma \ll 1$, one can write $\sigma\varphi \ll M_p$ which means that the second term in the first parentheses can be neglected. In this case, (32) gives for $l_3 = 2$,

$$\varphi_{min} = \frac{\beta\rho_m/M_p}{2V_0 - l_2\beta\rho_m/M_p^2} \quad (33)$$

This is a local minimum if the following condition is satisfied

$$V''_{eff}(\varphi_{min}) = 2V_0 - \frac{l_2\beta}{M_p^2}\rho_m > 0 \quad (34)$$

If we combine (33) and (17) we obtain

$$\frac{\Delta r_c}{r_c} = \frac{V_0}{3\Phi_c\beta^2 l_2^2} \frac{M_p^2}{\rho_{out}} \quad (35)$$

where we have used that facts that $\rho_{out} \ll \rho_{in}$ and $M_p^2 \ll \rho_{in}$ and ρ_{out} . For Earth to have a thin-shell, (35) takes the form

$$\frac{\Delta R_e}{R_e} = \frac{1}{3\Phi_e\beta^2 l_2^2} \frac{M_p^2}{\rho_a} \quad (36)$$

in which V_0 is taken to be of order of unity. The bound on the PPN parameter γ then reads[†]

$$|l_2(\sigma - l_2)| > 10^{-12} \quad (37)$$

Let us compare the latter constraint with (34). We first note that $\rho_{out}/M_p^2 \sim 10^{36} \text{ cm}^2$ and $\rho_{in}/M_p^2 \sim 10^{40} \text{ cm}^2$. For $V_0 \sim 1$, (34) implies that $l_2(\sigma - l_2) < 10^{-36}$ or 10^{-40} which is not consistent with (37). It is also possible to translate (34) into a constraint on the scale introduced by V_0 for reasonable values of l_2 . In this case, (34) and (37) still remain inconsistent.

[†]Here, we assume that l_2 is not much greater than unity.

[‡]We have used $M_p^2/\rho_a \sim 10^{-36} \text{ cm}^{-2}$.

3 Conclusions

In chameleon BD gravity, the BD scalar field is allowed to couple with matter sector via an arbitrary coupling function. One then pre-assumes that the scalar field is a chameleon in the sense that it can hide the anomalous coupling via chameleon mechanism and pass local gravity experiments. In this note, we have checked viability of this pre-assumption in some extent. To do this, we have written the model in the Einstein conformal frame. In Einstein frame representation, the matter system couples with the scalar field via two different functions, one exponential function which is given by the conformal transformation and the other $f(\varphi)$ which is also assumed to be an exponential function parameterized by l_2 . We have considered conditions that the whole anomalous coupling is suppressed by the chameleon mechanism. When $V(\varphi) = e^{l_1\varphi/M_p}$, the thin-shell condition for the earth gives $l_2 \sim \omega^{-1/2}$. On the other hand, the equivalence principle sets a lower bound $l_1 > 10^{16}$ which implies that the local gravity experiments can not be suppressed for reasonable values of the exponent. We have also examine power law potentials $V(\varphi) = V_0\varphi^{l_3}$. For a quadratic potential ($l_3 = 2$), we have obtained a certain condition on the parameter l_2 for which the potential has a local minimum. We have shown that this condition is not consistent with the thin-shell condition for the earth. These results create strong debates over viability of the pre-assumed chameleon behavior of the scalar field in the context of chameleon BD gravity models. This behavior seems to depend significantly on the potential and the coupling functions.

References

- [1] T. P. Sotiriou and V. Faraoni, Rev. Mod. Phys. **82**, 451 (2010)
- [2] C. Brans and R. H. Dicke, Phys. Rev. **124**, 925 (1961)
- [3] N. Banerjee and D. Pavon, Phys. Rev. D **63**, 043504 (2001)
- [4] C.M. Will, Theory and Experiment in Gravitational Physics, (Cambridge University Press, 3rd edition, Cambridge, 1993)
C. M. Will, Liv. Rev. Rel. **9**, 3 (2005)
- [5] O. Bertolami and P. J. Martins, Phys. Rev. D **61**, 064007 (2000)
M. K. Mak and T. Harko, Europhys. Lett. **60**, 155 (2002)
- [6] W. Chakraborty and U. Debnath, arxiv:0807.1776v1
- [7] S. Das and N. Banerjee, Phys. Rev. D **78**, 043512 (2008)
- [8] H. Farajollahi, M. Farhoudi, A. Salehi and H. Shojaie, Astrophys. Space Sci. **337**, 415 (2012)
H. Farajollahi and A. Salehi, JCAP **1011**, 006 (2010)
H. Farajollahi and A. Salehi, JCAP **07**, 036 (2011)
K. Saaidi, A. Mohammadi and H. Sheikahmadi, Phys. Rev. D **83**, 104019 (2011)

- [9] Y. Bisabr, Phys. Rev. D **86** 127503 (2012)
- [10] V. Faraoni, E. Gunzig and P. Nardone, Fund. Cosmic Phys. **20**, 121 (1999)
- [11] V. Faraoni, *Cosmology in Scalar-Tensor Gravity* (Kluwer Academic Publishers 2004)
- [12] B. Schutz, Phys. Rev. D **2** 2762 (1970)
J. D. Brown, Class. Quant. Grav. **10**, 1579 (1993)
- [13] S. W. Hawking and G. F. R. Ellis, *The Large Scale Structure of Space-Time* (Cambridge 1973, Cambridge University Press)
- [14] O. Bertolami, C. G. Bohmer, T. Harko and F. S. N. Lobo, Phys. Rev. D **75**, 104016 (2007)
S. Nojiri, S. D. Odintsov and P. V. Tretyakov, Prog. Theor. Phys. Suppl. **172**, 81 (2008)
- [15] O. Bertolami and J. Paramos, J. Phys: Conf. Ser. 222. 012010
O. Bertolami, P. Frazao and J. Paramos, Phys. Rev. D **83**, 044010 (2011)
O. Bertolami and A. Martins, Phys. Rev. D **85**, 024012 (2012)
- [16] T. P. Sotiriou and V. Faraoni, Class. Quant. Grav. **25**, 205002 (2008)
- [17] V. Faraoni, Phys. Rev. D **80**, 124040 (2009)
- [18] J. Khoury and A. Weltman, Phys. Rev. D **69**, 044026 (2004)
J. Khoury and A. Weltman, Phys. Rev. Lett. **93**, 171104 (2004)
- [19] T. Faulkner, M. Tegmark, E. F. Bunn and Yi Mao, Phys. Rev. D **76**, 063505 (2007)
- [20] S. Weinberg, *Gravitation and Cosmology*, John Wiley and Sons 1972